MATHEMATICS IN NATURE



This guide links the *Mathematics in Nature* unit to the Texas Essential Knowledge and Skills (TEKS) for third graders. *Mathematics in Nature* is a science and mathematics unit that allows students to explore and gain knowledge about mathematical patterns found in nature, such as tessellations and the Fibonacci sequence. The unit also has interdisciplinary connections to other subject areas. For example, students will communicate clearly by putting thoughts and feelings into spoken words, as covered in the English Language Arts and Reading TEKS. They will also understand the concepts of time and chronology, which the Social Studies TEKS cover. The following document includes the applicable TEKS and the details of the *Mathematics in Nature* unit. The asterisks indicate the TEKS that are testable on the State of Texas Assessments of Academic Readiness (STAAR). The final section of this document presents the applicable Texas College and Career Readiness Standards adopted by the Texas Higher Education Coordinating Board (THECB) on January 24, 2008.

Description of Unit

For this project, students will examine mathematical patterns found in nature, such as tessellations, the Fibonacci sequence, the golden ratio, and pi. For example, the students can create a tessellation jigsaw puzzle. Each student draws a tessellation pattern on a sheet of paper, cuts it out, jumbles up the pieces, and passes the pieces to another classmate for reassembly. In this way, the student looks at tessellations from two different perspectives: once, when constructing his or her own, and from a different perspective when putting together a classmate's tessellation jigsaw puzzle. Similarly, Fibonacci puzzles can be constructed using hexagons as in a beehive, or bricks as in a wall. Examples of these types of puzzles can be found at http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibpuzzles.html.

A student could make the pieces, write the list of rules for solving the puzzle, and pass the pieces with their rules on to another student for reassembly.



Goals

Students will meet these goals in their explorations:

- Ask questions and explore theories
- Have opportunities to generate new ideas
- Develop the essential skills of logical thinking, creative problem solving, intellectual risk taking, and communicating
- Become familiar with various patterns in nature and where they occur
- Understand the details of how and why such a pattern is constructed and which mathematical concept it illustrates

Phase I. Learning Experiences

1. Introduce the concepts of patterns found in nature, such as tessellations, the Fibonacci sequence, the golden ratio, and pi. Talk about where such patterns might be found. Show examples and have students bring other examples. Helpful websites include:

http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html http://mathforum.org/dr.math/faq/faq.golden.ratio.html http://www.goldenmeangauge.co.uk/fibonacci.htm http://megsl.org/pi.html http://42explore.com/teslatn.htm http://www.imdb.com/title/tt0052751/

- **2.** As a class, choose a pattern in nature, such as a pine cone, pineapple, turtle shell or starfish, that you all find interesting. Discuss its origin, how it is formed and why, and any special characteristics unique to that pattern. Which mathematical concept does it demonstrate?
- **3.** Discuss the various types of symmetry. Helpful information can be found at <u>http://www.teachersnetwork.org/dcs/math/symmetry/</u>.
- **4.** Divide the class into small groups. Each group chooses a pattern and breaks it down into its components. Each group will discuss whether their pattern is symmetrical, how it is formed in nature, examples of the pattern in nature, and which mathematical concept it demonstrates.
- 5. Each group presents their pattern and their findings to the class.

Phase II. Independent Research

A. Research process

- **1.** Selecting a topic. Each student should identify a natural pattern for more in-depth study.
- **2.** Asking guiding questions. Once students have selected their pattern, each student should think of three to five guiding questions, such as:





- Where is this pattern found in nature?
- What causes it to form that way?
- What are the pattern's units (i.e., what does each piece of the pattern look like and how does it function in nature?)
- Is the pattern symmetrical?
- Which mathematical concept does the pattern demonstrate?

While these examples are general, the student's questions should be specific to the chosen topic. The questions should lead him/her to form individual research-based opinions. The student should also develop a hypothesis or some possible answers to the questions.

- **3.** Designing a research proposal. The student should include numerous components in the research proposal:
 - The type of pattern he/she will study
 - Three to five guiding questions he/she will investigate, as well as hypothetical answers to those questions
 - Resources he/she will need to find answers to questions, such as primary and secondary sources, correspondence with experts on the subject, etc.
- 4. Conducting the research. After you have approved student proposals, each student begins using the resources he/she has identified and others he/she may encounter. During this stage, the student will need to keep a log, note cards, or resource process sheets of all the sources and what he/she has learned from each one.
- 5. Drawing conclusions. Based on the research, students should work together to plan a class fair for other students in the school or people in the community. Each student will create an activity or display for the fair, exhibiting a natural pattern that they have explored.

B. The product

The teacher may wish to arrange a class fair in which other students from the school come to learn about symmetry and nature. Each student creates an activity or a display with an interactive component to illustrate the natural pattern studied.

C. Communication

The student will present his/her activity or display at the fair. He/she should provide some background—why that activity or display was developed—and offer any solutions or findings. The audience should be given time for questions and answers. The Q&A session should be impromptu and unscripted in order to accurately reflect student learning.

D. A completed project consists of:

- 1. A research proposal, including guiding questions and answers
- 2. A research log, notes, or resource process sheets
- 3. The product—the activity or display presented at the class fair
- 4. A Works Cited Page
- 5. A videotape or audiotape of the student's talk, including the unscripted Q&A session



Texas Essential Knowledge and Skills

The unit may address the following TEKS:

English	Language Arts and Reading:	
3.1	Uses the relationships between letters and sounds, spelling patterns, and morphological analysis to decode written English	
3.2	Comprehends a variety of texts drawing on useful strategies as needed* (Testable on the Grade 3 Reading STAAR, Reporting Category 2)	
3.3	Reads grade-level text with fluency and comprehension	
3.4	Understands new vocabulary and uses it when reading and writing* (Testable on the Grade 3 Reading STAAR, Reporting Category 1)	
3.5	Analyzes, makes inferences, and draws conclusions about theme and genre in different cultural, historical, and contemporary contexts and provides evidence from the text to support their understanding* (Testable on the Grade 3 Reading STAAR, Reporting Category 2)	
3.12	Analyzes, makes inferences, and draws conclusions about the author's purpose in cultural, historical, and contemporary contexts and provides evidence from the text to support their understanding* (Testable on the Grade 3 Reading STAAR, Reporting Category 3)	
3.13	Analyzes, makes inferences, and draws conclusions about expository text and provides evidence from text to support their understanding* (Testable on the Grade 3 Reading STAAR, Reporting Category 3)	
3.15	Understands how to glean and use information in procedural texts and documents* (Testable on the Grade 3 Reading STAAR, Reporting Category 3; Testable on the Grade 4 Writing STAAR, Reporting Category 1, Reporting Category 2, Reporting Category 3)	
3.16	Uses comprehension skills to analyze how words, images, graphics, and sounds work together in various forms to impact meaning* (Testable on the Grade 3 Reading STAAR, Reporting Category 2, Reporting Category 3)	
3.17	Uses elements of the writing process (planning, drafting, revising, editing, and publishing) to compose text* (Testable on the Grade 4 Writing STAAR, Reporting Category 1)	
3.20	Writes expository and procedural or work-related texts to communicate ideas and information to specific audiences for specific purposes* (Testable on the Grade 4 Writing STAAR, Reporting Category 3)	
3.25	Asks open-ended research questions and develops a plan for answering them	
3.26	Determines, locates, and explores the full range of relevant sources addressing a research question and systematically record the information they gather	
3.29	Uses comprehension skills to listen attentively to others in formal and informal settings	
3.30	Speaks clearly and to the point, using the conventions of language	
Mathematics:		
3.1	Uses mathematical processes to acquire and demonstrate mathematical understanding	
3.4	Applies mathematical process standards to develop and use strategies and methods for whole	

number computations in order to solve problems with efficiency and accuracy

Mathematics in Nature (Grade 3)

- 3.6 Applies mathematical process standards to analyze attributes of two-dimensional geometric figures to develop generalizations about their properties
- 3.7 Applies mathematical process standards to select appropriate units, strategies, and tools to solve problems involving customary and metric measurement
- 3.8 Applies mathematical process standards to solve problems by collecting, organizing, displaying, and interpreting data

Science:

- 3.1 Conducts classroom and outdoor investigations following school and home safety procedures and environmentally appropriate practices* (Testable on the Grade 5 Science STAAR)
- 3.2 Uses scientific inquiry methods during laboratory and outdoor investigations* (Testable on the Grade 5 Science STAAR)
- 3.3 Knows that information, critical thinking, scientific problem solving, and the contributions of scientists are used in making decisions* (Testable on the Grade 5 Science STAAR)
- 3.4 Knows how to use a variety of tools and methods to conduct science inquiry* (Testable on the Grade 5 Science STAAR)
- 3.6 Knows that forces cause change and that energy exists in many forms* (Testable on the Grade 5 Science STAAR, Reporting Category 2)
- 3.8 Knows there are recognizable patterns in the natural world and among objects in the sky* (Testable on the Grade 5 Science STAAR, Reporting Category 3)
- 3.9 Knows that organisms have characteristics that help them survive and can describe patterns, cycles, systems, and relationships within the environments* (Testable on the Grade 5 Science STAAR, Reporting Category 4)
- 3.10 Knows that organisms undergo similar life processes and have structures that help them survive within their environments* (Testable on the Grade 5 Science STAAR, Reporting Category 4)

Social Studies:

- 3.3 Understands the concepts of time and chronology
- 3.5 Understands the concepts of location, distance, and direction on maps and globes
- 3.17 Applies critical-thinking skills to organize and use information acquired from a variety of valid sources, including electronic technology
- 3.18 Communicates in written, oral, and visual forms
- 3.19 Uses problem-solving and decision-making skills, working independently and with others, in a variety of settings



Texas College and Career Readiness Standards

This unit may address the following Texas College and Career Readiness Standards:

English Language Arts:		
I.A.1	Determines effective approaches, forms, and rhetorical techniques that demonstrate understanding of the writer's purpose and audience	
I.A.2	Generates ideas and gathers information relevant to the topic and purpose, keeping careful records of outside sources	
I.A.3	Evaluates relevance, quality, sufficiency, and depth of preliminary ideas and information, organizes material generated, and formulate thesis	
II.A.1	Uses effective reading strategies to determine a written work's purpose and intended audience	
II.A.2	Uses text features and graphics to form an overview of informational texts and to determine where to locate information	
II.A.3	Identifies explicit and implicit textual information, including main ideas and author's purpose	
II.A.4	Draws and supports complex inferences from text to summarize, draw conclusions, and distinguish facts from simple assertions and opinions	
II.A.8	Compares and analyzes how generic features are used across texts	
II.A.9	Identifies and analyzes the audience, purpose, and message of an informational or persuasive text	
II.B.1	Identifies new words and concepts acquired through study of their relationships to other words and concepts	
III.A.1	Understands how style and content of spoken language varies in different contexts and influences the listener's understanding	
III.A.2	Adjusts presentation (delivery, vocabulary, length) to particular audiences and purposes.	
III.B.1	Participates actively and effectively in one-on-one oral communication situations	
III.B.2	Participates actively and effectively in group discussions	
III.B.3	Plans and delivers focused and coherent presentations that convey clear and distinct perspectives and demonstrate solid reasoning	
IV.A.1	Analyzes and evaluates the effectiveness of a public presentation	
IV.A.2	Interprets a speaker's message; identifies the position taken and the evidence in support of that position	
IV.A.3	Uses a variety of strategies to enhance listening comprehension	
IV.B.1	Listens critically and responds appropriately to presentations	
IV.B.2	Listens actively and effectively in one-on-one communication situations	
IV.B.3	Listens actively and effectively in group discussions	
V.A.1	Formulates research questions	

Mathematics in Nature (Grade 3)



Explores a research topic		
Refines research topic and devise a timeline for completing work		
Gathers relevant sources		
Evaluates the validity and reliability of sources		
Synthesizes and organizes information effectively		
Uses source material ethically		
Designs and presents an effective product		
Mathematics:		
Identifies and applies transformations to figures		
Identifies the symmetries of a plane figure		
Uses congruence transformations and dilations to investigate congruence, similarity, and symmetries of plane figures		
Applies probabilistic measures to practical situations to make an informed decision		
Plans a study		
Determines types of data		
Selects and applies appropriate visual representations of data		
Describes patterns and departure from patterns in a set of data		
Analyzes given information		
Formulates a plan or strategy		
Determines a solution		
Justifies the solution		
Evaluates the problem-solving process		
Develops and evaluate convincing arguments		
Uses various types of reasoning		
Formulates a solution to a real-world situation based on the solution to a mathematic problem		
Uses a function to model a real-world situation		
Evaluates the problem-solving process		
Uses mathematics as a language for reasoning, problem solving, making connections, and generalizing		
Communicates mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and words		
Creates and uses representations to organize, record, and communicate mathematical ideas		
Explains, displays, or justifies mathematical ideas and arguments using precise mathematical language in written or oral communications		
Connects and uses multiple strands of mathematics in situations and problems		
Connects mathematics to the study of other disciplines		
Uses multiple representations to demonstrate links between mathematical and real-world		

Mathematics in Nature (Grade 3)

situations X.B.2 Understands and uses appropriate mathematical models in the natural, physical, and social sciences Science: I.A.2 Uses creativity and insight to recognize and describe patterns in natural phenomena. I.C.1 Collaborate on joint projects I.E.1 Uses several modes of expression to describe or characterize natural patterns and phenomena These modes of expression include narrative, numerical, graphical, pictorial, symbolic, and kinesthetic I.E.2 Uses essential vocabulary of the discipline being studied III.B.1 Reads technical and scientific articles to gain understanding of interpretations, apparatuses, techniques or procedures, and data III.B.2 Sets up apparatuses, carries out procedures, and collects specified data from a given set of appropriate instructions Recognizes scientific and technical vocabulary in the field of study and use this vocabulary to III.B.3 enhance clarity of communication III.B.4 Lists, uses, and gives examples of specific strategies before, during, and after reading to improve comprehension III.C.1 Prepares and represents scientific/technical information in appropriate formats for various audiences III.D.1 Uses search engines, databases, and other digital electronic tools effectively to locate information III.D.2 Evaluates quality, accuracy, completeness, reliability, and currency of information from any source V.C.1 Recognizes patterns of change V.D.1 Understands that scientists categorize things according to similarities and differences **V.E.1** Uses models to make predictions **Social Studies:** I.F.1 Uses a variety of research and analytical tools to explore questions or issues thoroughly and fairly IV.A.3 Evaluates sources from multiple perspectives IV.A.4 Understands the differences between a primary and secondary source and uses each appropriately to conduct research and construct arguments IV.A.5 Reads narrative texts critically IV.A.6 Reads research data critically IV.B.1 Uses established research methodologies

- IV.B.2 Explains how historians and other social scientists develop new and competing views of past phenomena
- IV.B.3 Gathers, organizes, and displays the results of data and research



- IV.B.4 Identifies and collects sources
- IV.C.1 Understands/interprets presentations critically
- IV.D.1 Constructs a thesis that is supported by evidence
- V.A.1 Uses appropriate oral communication techniques depending on the context or nature of the interaction
- V.A.2 Uses conventions of standard written English
- V.B.1 Attributes ideas and information to source materials and authors

Cross-Disciplinary Standards:

- I.A.1 Engages in scholarly inquiry and dialogue
- I.B.2 Constructs well-reasoned arguments to explain phenomena, validate conjectures, or support positions
- I.B.3 Gathers evidence to support arguments, findings, or lines of reasoning
- I.B.4 Supports or modifies claims based on the results of an inquiry
- I.C.1 Analyzes a situation to identify a problem to be solved
- I.C.2 Develops and applies multiple strategies to solving a problem
- I.D.1 Self-monitors learning needs and seeks assistance when needed
- I.D.2 Uses study habits necessary to manage academic pursuits and requirements
- I.D.3 Strives for accuracy and precision
- I.D.4 Perseveres to complete and master tasks
- I.E.1 Works independently
- I.E.2 Works collaboratively
- I.F.1 Attributes ideas and information to source materials and people
- I.F.2 Evaluates sources for quality of content, validity, credibility, and relevance
- II.A.1 Uses effective prereading strategies
- II.A.2 Uses a variety of strategies to understand the meanings of new words
- II.A.3 Identifies the intended purpose and audience of the text
- II.A.4 Identifies the key information and supporting details
- II.A.5 Analyzes textual information critically
- II.A.6 Annotates, summarizes, paraphrases, and outlines texts when appropriate
- II.A.7 Adapts reading strategies according to structure of texts
- II.B.1 Writes clearly and coherently, using standard writing conventions
- II.B.2 Writes in a variety of forms for various audiences and purposes
- II.B.3 Composes and revises drafts
- II.C.1 Understands which topics or questions are to be investigated
- II.C.2 Explores a research topic
- II.C.3 Refines a research topic based on preliminary research and devise a timeline for completing work
- II.C.4 Evaluates the validity and reliability of sources

- II.C.5 Synthesizes and organizes information effectively
- II.C.6 Designs and presents an effective product
- II.C.7 Integrates source material
- II.C.8 Presents final product
- II.D.1 Identifies patterns or departures from patterns among data
- II.D.2 Uses statistical and probabilistic skills necessary for planning an investigation and collecting, analyzing, and interpreting data
- II.D.3 Presents analyzed data and communicate findings in a variety of formats
- II.E.1 Uses technology to gather information
- II.E.2 Uses technology to organize, manage, and analyze information
- II.E.3 Uses technology to communicate and display findings in a clear and coherent manner
- II.E.4 Uses technology appropriately



Mathematics in Nature (Grade 3)